9/24/19

MLS4: UDD & CF

x - integer
$0 \le t \le 1$

UDD: ${}_tq_x = t \cdot q_x$

$$l_{x+t} = (1-t) \cdot l_x + t \cdot l_{x+1}$$

CF: ${}_tp_x = p^t$

$$l_{x+t} = l_x^{1-t} \cdot l_{x+1}^{t}$$

MLS4 Exercises:

6) UDD $q_0 = 0.15$, $q_1 = 0.10$, $q_2 = 0.05$? (a) ${}_tP_1$ & (b) ${}_tP_2$

Method 1: (Easier Way)

${}_tP_1 = 1 - {}_tq_1 \overset{UDD}{=} 1 - t \cdot q_1 = 1 - .1t$

${}_tP_2 = 1 - {}_tq_2 \overset{UDD}{=} 1 - t \cdot q_2 = 1 - .05t$

Method 2: (for (b); could also use for (a))

${}_tP_2 = \frac{l_{2+t}}{l_2}$ $\quad l_{2+t} = (1-t)l_2 + t \cdot l_3$

Set $l_2 = 1000$ (arbitrary)

$\Rightarrow l_3 = l_2 \cdot P_2 = 1000(.95) = 950$

$\therefore l_{2+t} = (1-t) \cdot 1000 + t \cdot 950 = 1000 - 50t$

$\therefore {}_tP_2 = \frac{1000 - 50t}{1000} = 1 - .05t$

7) $l_{30} = 1000$ $\quad ?\ {}_{30}p_{35} = \dfrac{l_{65}}{l_{35}}$

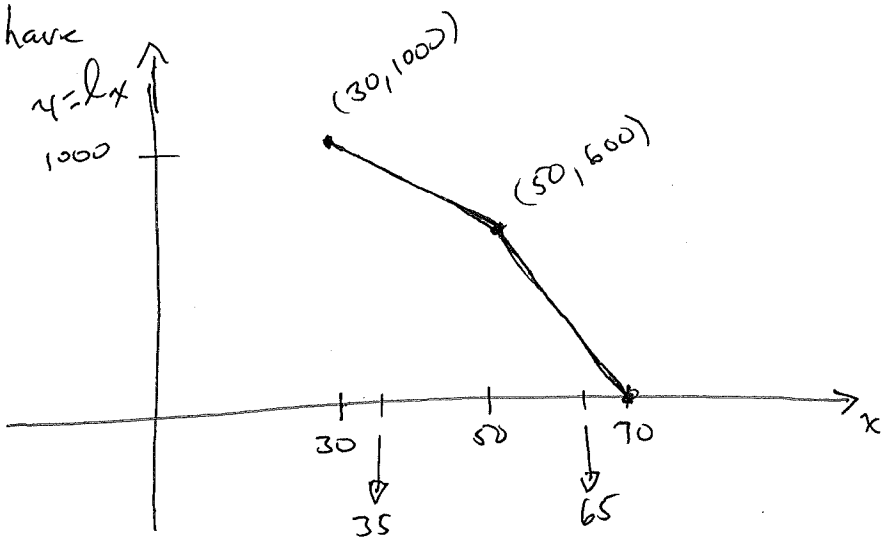
$_nd_{30} = 20n \quad 0 \le n \le 20 \Rightarrow$ 20 deaths/year between ages 30 & 50

$_nd_{50} = 30n \quad 0 \le n \le 20 \Rightarrow$ 30 deaths/year between ages 50 & 70

(UDD type problem)

$$\left.\begin{aligned} l_{30} &= 1000 \\ {}_{20}d_{30} &= 400 \end{aligned}\right\} \Rightarrow l_{50} = 600 \left.\begin{aligned} \\ {}_{20}d_{50} &= 600 \end{aligned}\right\} \Rightarrow l_{70} = 0$$

We have



$$l_{35} = \frac{15}{20} l_{30} + \frac{5}{20} l_{50} = \frac{15}{20}(1000) + \frac{5}{20}(600) = 900$$

$$l_{65} = \frac{5}{20} l_{50} + \frac{15}{20} l_{70} = \frac{5}{20}(600) + \frac{15}{20}(0) = 150$$

$$\therefore\ {}_{30}p_{35} = \frac{150}{900} = \frac{1}{6}$$

8) ${}_tP_{30} = (.9)^t \quad 0 \le t \le 20$? ${}_{30}P_{35}$

${}_tP_{50} = (.8)^t \quad 0 \le t \le 20$

CF problem

$${}_{30}P_{35} = {}_{15}P_{35} \cdot {}_{15}P_{50}$$

$$= (.9)^{15} \cdot (.8)^{15}$$

Remark: ~~${}_{20}P_{30}$~~ ${}_{20}P_{30} = {}_5P_{30} \cdot {}_{15}P_{35}$

$$\Rightarrow {}_{15}P_{35} = \frac{{}_{20}P_{30}}{{}_5P_{30}} = \frac{(.9)^{20}}{(.9)^5} = .9^{15}$$

MISS: Select & Ultimate Rates

(x): a person age x (in general population)

[x]: a person age x (selected from the general pop.)

(x+1)

[x+1]

[x]+1